NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BSOC; 07BSAM	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME**: LINEAR ALGEBRA 2
SESSION: JANUARY 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS: 100**

SUPPLEMENTARY / SECOND OPPORTUNITY EXAMINATION PAPER	
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INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [36]

Let V and W be vectors spaces over a filed $\mathbb{R}$ and $T: V \to W$ be a mapping.

1.1. State what does it means to say T is linear. [3]

1.2. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} y+z \\ z \\ x-z \end{bmatrix}$.

(a) Show that T is linear. [14]

(b) Find the matrix of T with respect to the standard basis of $\mathbb{R}^3$. [7]

(c) Use the result in (b) to find the Characteristic polynomial of T. [5]

1.3. Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be given by T(x, y , z) = (|x|, y + z). Determine whether T is linear on not.

[7]

QUESTION 2 [23]

2.1. Let $\mathcal{B} = \{v_1, v_2\}$ and $C = \{u_1, u_2\}$ be bases for a vector space V and suppose

$v_1 = 6u_1 - 2u_2$ and $v_2 = 9u_1 - 4u_2$.

(a) Find the change of coordinate matrix from $\mathcal{B}$ to C. [5]

(b) Use part (a) to find $[x]_C$ for $x = -3v_1 + 2v_2$. [5]

2.2. In P_2, find the change-of-coordinates matrix from the basis

$\mathcal{B} = \{1 - 2t + t^2, 3 + 4t^2,\ 2t + 3t^2\}$ to the standard basis $S = \{1, t, t^2\}$. [5]

2.3. Let $\mathcal{B} = \{v_1, v_2, v_3\}$ be a basis of $\mathbb{R}^3$ in which $v_1 = (1,1,0), v_2 = (0,1,2)$ and

$v_3 = (1,0,-1)$. Find the coordinate vector of $v = (1,2,3)$ with respect to the basis $\mathcal{B}$. [8]

QUESTION 3 [8]

Let $A = PDP^{-1}$ where $P = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$. Then Compute A^{10}.

QUESTION 4 [10]

Find the quadratic form q(X) that corresponds to the symmetric matrix

$$\begin{bmatrix} 0 & 4 & 2 \\ 4 & 1 & 3 \\ 2 & 3 & -2 \end{bmatrix}.$$ [10]

QUESTION 5 [23]

5.1. Is $v = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ an eigenvector of $A = \begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$? If so, find the corresponding eigenvalue. [6]

5.2. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & -2 \\ 3 & 0 & 3 \end{bmatrix}$. Find the eigenvalues of A and the eigenspace corresponding to the largest eigenvalue. [17]

END OF SUPPLEMENTARY / SECOND OPPORTUNITY EXAMINATION QUESTION PAPER